

Joint Spectrum Reservation and On-demand Request for Mobile Virtual Network Operators

Yingxiao Zhang, Suzhi Bi, and Ying Jun (Angela) Zhang

Abstract

With wireless network virtualization, Mobile Virtual Network Operators (MVNOs) can develop new services on a low-cost platform by leasing virtual resources from mobile network owners. In this paper, we investigate a two-stage spectrum leasing framework, where an MVNO acquires radio spectrum through both advance reservation and on-demand request. To maximize its surplus, the MVNO jointly optimizes the amount of spectrum to lease in the two stages by taking into account the traffic distribution, random user locations, wireless channel statistics, Quality of Service (QoS) requirements, and the prices differences. Meanwhile, the acquired spectrum resources are dynamically allocated to the MVNO's mobile subscribers (users) according to fast channel fadings in order to maximize the utilization of the resources. The MVNO's surplus maximization problem is naturally formulated as a tri-level nested optimization problem that consists of Dynamic Resource Allocation (DRA), on-demand request, and advance reservation subproblems. To solve the problem efficiently, we rigorously analyze the structure of the optimal solution in the DRA problem, and the optimal value is used to find the optimal leasing decisions in the two stages. In particular, we derive closed-form expressions of the optimal advance reservation and on-demand requests when the proportional fair utility function is adopted. We further extend the analysis to general utility functions and derive a Stochastic Gradient Decent (SGD) algorithm to find the optimal leasing decisions. Simulation results show that the two-stage spectrum leasing strategy can take advantage of both the price discount of advance reservation and the flexibility of on-demand request to deal with traffic variations.

Index Terms

radio spectrum management, mobile virtual network operator, optimization

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I. INTRODUCTION

Wireless Network Virtualization (WNV) is an emerging technology that provides unprecedented opportunities for Mobile Virtual Network Operators (MVNOs) to develop new services at a low cost by leasing infrastructure and radio resources from Mobile Network Owners (MNOs) [1]–[3]. In contrast to the virtualization of network resources in wired networks, the virtualization of radio spectrum is a unique problem in WNV due to the broadcasting and stochastic nature of wireless channels. Unlike other network resources, radio spectrum can be dynamically reused by different links based on the geographic separation of transmission nodes, transmit powers, the interference cancellation capability, and the Quality of Service (QoS) requirements. As a result, spectrum virtualization is much more complicated and deserves in-depth study.

Most previous work on radio spectrum virtualization focuses on resource slicing from the MNO's perspective (see [4] and the references therein). In contrast, much less attention has been paid to the MVNO's optimal strategies in terms of determining when and how much spectrum resources to ask for. The limited existing work on spectrum leasing from the MVNO's perspective can be divided into two threads, i.e., long-term advance reservation [5], [6] and short-term on-demand request [7]–[9]. In particular, in [5], [6], the MVNO acts as a retailer that buys spectrum in advance from the MNO at a discount wholesale price and attracts end mobile users by advertising and service differentiation. However, due to the uncertainty in traffic demands and wireless channel conditions, under-reservation (over-reservation) may occur when the reserved resources is less (more) than that of the real-time demand. This problem can be solved by on-demand request which allows the MVNO to dynamically purchase spectrum according to the real-time demand. In [7]–[9], the spectrum leasing decisions vary very fast according to not only the number of users and their locations, but also the fast channel fadings. In general, on-demand request is more expensive than advance reservation due to the higher cost on frequent trading and resource reallocation. Moreover, adapting spectrum leasing decisions with respect to fast fadings is too costly for practical implementation.

In contrast to the conventional one-stage spectrum leasing, this paper advocates a two-stage spectrum leasing scheme where the MVNO obtains spectrum from both advance reservation and on-demand request. When carefully optimized, the two-stage leasing scheme enjoys the complementary strengths of the two stages. The first stage, advance reservation, reduces risks and operation complexity for both MVNOs and MNOs. On one hand, advance reservation ensures

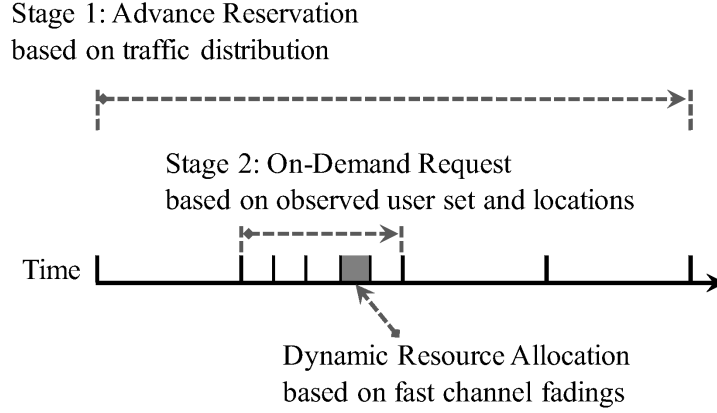


Fig. 1: Illustration of the two-stage spectrum leasing framework.

MVNOs a baseline amount of spectrum at a relatively low cost. On the other hand, it allows MNOs to pre-plan spectrum slicing and system operation in an early stage. Meanwhile, the second stage, on-demand request, preserves flexibility and competition. On one hand, MVNOs have the flexibility to acquire additional spectrum according to the real-time traffic demand, and thus avoid being overly conservative in the first stage. On the other hand, the MNO can derive more profit by setting a higher on-demand price according to the real-time competition level among multiple MVNOs. The two-stage leasing framework has been previously considered in cloud networks to serve the uncertain demand of computing and storage resources [10], [11]. However, in WNV, random wireless channel fading introduces a new dimension of uncertainty, rendering it difficult for an MVNO to anticipate its need of spectrum in advance.

In this paper, we aim to find the optimal two-stage spectrum leasing strategy of an MVNO in WNV. Specifically, the system operation involves three timescales, as depicted in Fig. 1. In the first advance reservation stage, the MVNO reserves a certain amount of spectrum for a long period of time, which usually covers hours or days. The decision is optimized according to the traffic statistics over the period. In the second on-demand request stage, the MVNO decides whether to request additional spectrum after observing the realization of actual number and locations of users it needs to serve. The on-demand request changes with user arrivals, departures and movements, and varies in a timescale of seconds. In contrast to [7], [9], the on-demand request here does not vary with fast channel fadings, and thus involves much lower complexity. Then, in the timescale of milliseconds, the MVNO dynamically allocates the spectrum resources acquired from both advance reservation and on-demand to the users based on fast channel fadings. To

maximize its profit, the MVNO needs to jointly optimize the operations in all three timescales.

The two-stage spectrum leasing problem is naturally formulated as a tri-level nested optimization problem that consists of Dynamic Resource Allocation (DRA), on-demand request, and advance reservation subproblems. Solving the problem is challenging in two aspects. First, the nested structure makes the three subproblems closely intertwined. The decision made in a larger timescale affects the optimization in a smaller timescale. In turn, the optimal value of a smaller-timescale problem is embedded in the objective function of a larger level problem. Thus, it is critical to analytically characterize the optimal values of the smaller-timescale problems, so that the larger-timescale problems are amenable to efficient solution algorithms. Secondly, the nested optimization problem is stochastic in the sense that the decision in a larger timescale must be optimized for random network realizations in a smaller timescale.

In this paper, we address the challenges as follows:

- We analytically characterize the optimal channel-aware DRA policy under a broad class of utility functions. Through rigorous analysis, we derive a closed-form expression of the optimal utility as a function of the total number of sub-carriers (SC) acquired from both the advance reservation and on-demand request stages.
- Based on the result from DRA, we analyze the optimal solutions of the on-demand request and advance reservation subproblems. In particular, when proportional fair (PF) utility is adopted, we derive closed-form expressions of the optimal number of SCs the MVNO should lease during the two stages. In other words, the MVNO can find the optimal operations in all three timescales by analytical calculations with negligible computational complexity.
- We extend the analysis to the case of general utility functions and derive a closed-form expression of the optimal solution in the on-demand request problem. Consequently, we develop a stochastic gradient descent (SGD) algorithm to solve the advance reservation problem, where the gradients are easily calculated from the closed-form solution derived from the on-demand request problem.

Our numerical results show that the proposed algorithm hits an optimal balance between advance reservation and on-demand request according to different levels of traffic and price variations. It significantly outperforms the conventional one-stage spectrum leasing schemes under various network conditions.

The rest of paper is organized as follows. In Section II, we describe the network model and introduce the two-stage spectrum leasing framework. In Section III, we solve the DRA problem

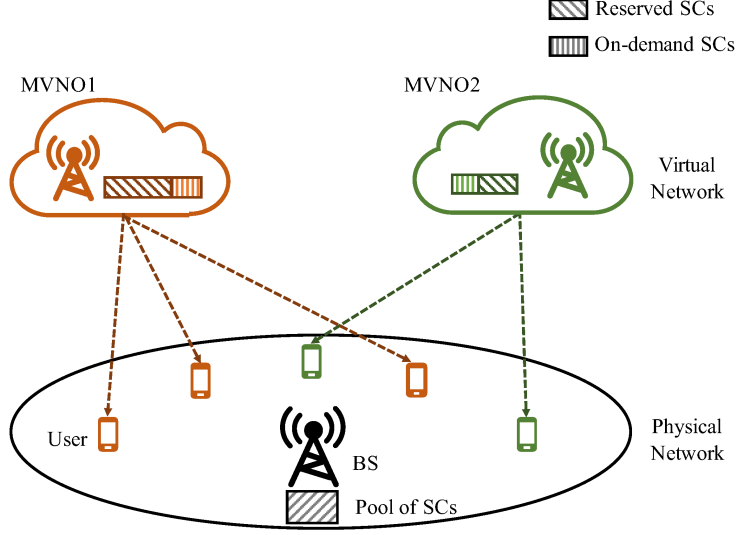


Fig. 2: A snapshot of the virtualized mobile network. Each MVNO allocates the acquired sub-carriers (SCs) from both advance reservation and on-demand request to its users.

for a general utility functions. In Section IV, we derive the optimal advance reservation and on-demand requests when PF utility is adopted. In Section V, we extend the analysis to general utility functions. The numerical results and discussions are presented in Section VI. Finally, we conclude this work in Section VII.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a single MVNO that acquires radio spectrum from an MNO and programs on the acquired spectrum to serve its users. In particular, we focus on the downlink transmission in a single OFDMA cell, where the spectrum owned by the MNO is divided into a number of Sub-Channels (SCs), each with bandwidth B . A snapshot of the mobile network is shown in Fig. 2. Without loss of generality, we assume that the cell is a circular area around the base station (BS) at the origin, denoted by $\mathcal{D}(o, D)$ where D is the cell radius. The set of users requesting services from the MVNO at the same time is denoted by a random point process $\mathcal{X} = \{x_k, \forall k = 1, \dots, K\}$, where each $x_k \in \mathcal{D}$ is the location of the k -th user, and $K \in \mathbb{Z}_+$ is total number of users. We assume that each x_k is uniformly distributed in the cell. The distribution of K is characterized by its probability mass function (PMF) $f_K(\cdot)$ in the domain $[K_a, K_b]$, where K_a is the lower bound and K_b is the upper bound. The set \mathcal{X} varies due to random user arrival, departure, and mobility. In this way, the traffic statistics is characterized by

the distribution of \mathcal{X} . Suppose that transmitted signals are affected by both large-scale path loss and fast Rayleigh fadings. The instantaneous data rate for the k -th user on the i -th SC is given by

$$b_{ki} = B \log \left(1 + \frac{P_t \ell(\|x_k\|) g_{ki}}{\Gamma N_0 B} \right), \quad (1)$$

where P_t denotes the fixed transmit power, $\ell(\cdot)$ denotes the path loss function, $\|\cdot\|$ denotes the Euclidean norm, g_{ki} denotes the power gain of the i.i.d. Rayleigh fading, N_0 denotes the power spectral density of white noise at the receiver, and Γ denotes the capacity margin [12].

The two-stage spectrum leasing scheme involves operations in three timescales, as shown in Fig. 1. For the sake of clarity, we define *a period*, *a session*, and *a time-slot* as three basic time units, during which traffic statistics (i.e., the distribution of \mathcal{X}), user locations (i.e., the realization of \mathcal{X}), and channel fadings remain unchanged, respectively. Typically, a period is measured in hours, a session in seconds, and a time-slot in milliseconds. At the beginning of a period, the MVNO reserves a number of SCs for the whole period according to the distribution of user set \mathcal{X} . Then, at the beginning of each session, the MVNO decides whether to lease additional SCs for the session according to the observed set of users \mathcal{X} . The acquired SCs from both reservation and on-demand request are dynamically allocated to the users according to fast channel fadings in each time-slot. The MVNO's surplus maximization problem can be formulated as a tri-level nested optimization problem, as explained in the following.

1) Advance Reservation: Let c_r be the reservation price of SCs, and n_r be the number of reserved SCs. The problem of finding the optimal reservation is formulated as a stochastic optimization problem:

$$\underset{n_r \in \mathbb{Z}_+}{\text{maximize}} \quad -c_r n_r + \mathbf{E}_{\mathcal{X}, c_s} [Q(\mathcal{X}, c_s, n_r)], \quad (2)$$

where the first term in the objective is the reservation cost, and the second term is the expected surplus from each session. The expectation is taken over all possible user realizations \mathcal{X} and the on-demand price c_s in a session. $Q(\mathcal{X}, c_s, n_r)$ denotes the surplus from a session given \mathcal{X} , c_s and n_r , and will be defined explicitly later. Notice that the reserved SCs cannot be released back to the MNO until the end of the period.

The reservation should take into account the traffic uncertainty from the user side, as well as the price uncertainty from the MNO side. In particular, the user set \mathcal{X} varies across sessions due to random user arrivals, departures, and mobility. Moreover, the on-demand price c_s varies according to the competition level among MVNOs in each session. We assume that the MVNO

can estimate the distribution of c_s during a period according to the historical information. The Cumulative Distribution Function (CDF) of c_s is denoted by $F_{c_s}(\cdot)$. The case with a constant on-demand price has been investigated in our previous work [13]. Typically, the MNO sets c_r lower than c_s in order to encourage reservation in advance. We will discuss the impact of the price differences in details through rigorous analysis in the following sessions.

2) *On-demand Request*: At the beginning of each session, the MVNO decides whether to request additional SCs according to the observed user set \mathcal{X} and the on-demand price c_s announced by the MNO. Let n_s be the requested number of SCs in the session. The problem of finding the optimal on-demand request is formulated as

$$Q(\mathcal{X}, c_s, n_r) = \underset{n_s \in \mathbb{Z}_+}{\text{maximize}} \quad -c_s n_s + u_g G(\mathcal{X}, n_r + n_s), \quad (3)$$

where u_g is a fixed scaler that converts the utility into monetary unit, and $G(\mathcal{X}, n)$ denotes the maximum utility achieved by serving the user set \mathcal{X} with n SCs. The first term in the objective of (3) is the cost of the on-demand SCs, and the second term is the utility-based income by serving the users. $Q(\mathcal{X}, c_s, n_r)$ is referred to as the surplus from each session that becomes part of the MVNO's total profit. Note that unlike the advance reservation, the on-demand request decisions are updated in each session.

3) *Dynamic Resource Allocation*: The maximum system utility $G(\mathcal{X}, n)$ of a session is a result of PHY-layer DRA. In particular, the MVNO dynamically allocates $n = n_r + n_s$ SCs to the K users in \mathcal{X} according to the fast channel fadings in each time-slot. Benefiting from the independent channel variations across users and SCs, DRA can substantially improve the system utility due to multiuser diversity. Let $U(\bar{r})$ denote the utility function of each user, where \bar{r} is the average throughput over the session. We assume $U(\bar{r})$ as a continuously differentiable increasing concave function which captures the satisfaction level of the user when received throughput \bar{r} [14]. By deploying different utility functions, the MVNO can balance the trade-off between spectral efficiency and fairness among users.

Let $g_{ki}[t]$ denote the fast channel fading between the BS and the k -th user on the i -th SC at the t -th time-slot, and $b_{ki}[t]$ denote the corresponding achievable data rate calculated by (1). Let $a_{ki}[t] \in [0, 1]$ denotes the fraction of airtime of the i -th SC allocated to the k -th user in the t -th time-slot, and $A = \{a_{ki}[t], \forall k, i, t\}$ denote all the allocation decisions in the session. Then, the

average throughput of the k -th user (for $k = 1, \dots, K$) is given by

$$\bar{r}_k = \frac{1}{T} \sum_{t=1}^T \sum_{i=1}^n a_{ki}[t] b_{ki}[t], \quad (4)$$

where T is the total number of time-slots in a session.

The problem of finding the optimal SC allocation for all time-slots can be formulated as:

$$G(\mathcal{X}, n) = \underset{A}{\text{maximize}} \sum_{k=1}^K U(\bar{r}_k) \quad (5a)$$

$$\text{subject to } \sum_{k=1}^K a_{ki}[t] = 1, \quad \forall i, t \quad (5b)$$

$$a_{ki}[t] \geq 0, \quad \forall k, i, t. \quad (5c)$$

Specially, $G(\emptyset, n) = 0$, meaning that the utility is 0 if the user set is empty. Notice that $G(\mathcal{X}, n)$ may be negative if $U(\bar{r})$ represents the penalties. For example, when $U(\bar{r}) = -1/\bar{r}$, the DRA problem in (5) tries to minimize the overall transmission delay.

From the description above, we can see that the three optimization problems are nested. The optimal values of the problems in the smaller timescales, i.e., $Q(\mathcal{X}, c_s, n_r)$ and $G(\mathcal{X}, n)$, are embedded in the objective functions in the larger timescales (2) and (3). In turn, the decisions in larger timescales n_r and n_s are parameters of the problem in smaller timescales. In what follows, we first study the DRA problem in (5) in Section III. Using the results from DRA, we derive the optimal reservation and on-demand request for the popular PF utility function in Section IV. We generalize the results to a wider class of utility functions in Section V.

III. DYNAMIC RESOURCE ALLOCATION

We first study the problem of DRA in (5), which tries to maximize the utilized of acquired SCs according to the fast channel fadings in each time-slot.

A. Optimal DRA Policy

Let $\Lambda = \{\lambda_i[t], \forall i, t\}$ and $\mathcal{V} = \{\nu_{ki}[t], \forall i, j, t\}$ denote the Lagrangian dual variables corresponding to (5b) and (5c), respectively. The Lagrange function is given by

$$L(A, \Lambda, \mathcal{V}) = \sum_{k=1}^K U(\bar{r}_k) + \sum_{i=1}^n \sum_{t=1}^T \lambda_i[t] \left(1 - \sum_{k=1}^K a_{ki}[t] \right) + \sum_{i=1}^n \sum_{k=1}^K \sum_{t=1}^T \nu_{ki}[t] a_{ki}[t]. \quad (6)$$

Let A^* denote the optimal solution and $\bar{\mathbf{r}}^* = [\bar{r}_1^*, \dots, \bar{r}_K^*]$ denote the corresponding optimal average throughputs for all users. The following KKT conditions hold:

$$\left. \frac{\partial L}{\partial a_{ki}[t]} \right|_{A^*} = \nabla U(\bar{r}_k^*) \frac{b_{ki}[t]}{T} - \lambda_i[t] + \nu_{ki}[t] = 0, \quad \forall k, i, t \quad (7a)$$

$$\sum_{k=1}^K a_{ki}^*[t] = 1, \quad \forall i, t \quad (7b)$$

$$a_{ki}^*[t] \geq 0, \quad \nu_{ki}[t] \geq 0, \quad \nu_{ki}[t] a_{ki}^*[t] = 0, \quad \forall k, i, t. \quad (7c)$$

Here, $\nabla U(r_0) = \frac{dU(\bar{r})}{d\bar{r}} \Big|_{\bar{r}=r_0}$ is the first-order derivative of $U(\bar{r})$ evaluated at $\bar{r} = r_0$.

We can infer from (7a) and (7c) that if $a_{ki}^*[t] > 0$, then $\nu_{ki}[t] = 0$ and

$$\nabla U(\bar{r}_k^*) b_{ki}[t] = T \lambda_i[t] \geq \nabla U(\bar{r}_{k'}^*) b_{k'i}[t], \quad \forall k' \neq k. \quad (8)$$

In other words, the i -th SC is only allocated to the user(user) that has(have) the largest $\nabla U(\bar{r}_k^*) b_{ki}[t]$. As $b_{ki}[t]$ is drawn from a continuous distribution, the probability that two or more users have the same value of $\nabla U(\bar{r}_k^*) b_{ki}[t]$ is zero. Therefore, each SC is exclusively allocated to a single user in each time-slot. The optimal allocation policy is described by:

$$a_{ki}^*[t] = \begin{cases} 1, & k = \arg \max_{k'} \nabla U(\bar{r}_{k'}^*) b_{k'i}[t] \\ 0, & \text{otherwise.} \end{cases} \quad (9)$$

The policy in (9) states that the SC in each time-slot is allocated to the user which can obtain a relatively high bit rate with respect to a function of its average throughput. Notice that the SC allocation policy in (9) requires the knowledge of the optimal throughputs $\bar{\mathbf{r}}^*$, which will be derived in the next subsection.

B. Optimal Average Throughput

With the SC allocation policy in (9), the average throughput of each user is given by

$$\bar{r}_k^* = \frac{1}{T} \sum_{t=1}^T \sum_{i=1}^n b_{ki}[t] \mathbf{1}_{\{\nabla U(\bar{r}_k^*) b_{ki}[t] > \nabla U(\bar{r}_{k'}^*) b_{k'i}[t], \forall k' \neq k\}}, \quad (10)$$

$\forall k = 1, \dots, K$, where $\mathbf{1}_{\{\cdot\}}$ is the indicator function with value 1 if the argument is true, and 0 otherwise. By the assumption that T is sufficiently large so that channel fadings are ergodic, the time-averaged throughput in (10) can be transferred to the expectation with respect to $\{b_{ki}, \forall k, i\}$, where b_{ki} denotes the random bit rate for the k -th user on the i -th SC. Moreover, as the fast channel fading for the same user on different SCs are identically distributed, b_{ki} 's for all i

follow a same distribution, where the CDF and Probability Density Function (PDF) are denoted by $F_{b_k}(\cdot)$ and $f_{b_k}(\cdot)$, respectively. The distribution functions can be obtained by the Rayleigh fading distribution according to (1), and only depend on the user location x_k . Then, (10) can be simplified as

$$\begin{aligned}
\bar{r}_k^* &= \sum_{i=1}^n \mathbf{E}_{\{b_{ki}\}} \left[b_{ki} \mathbf{1}_{\{\nabla U(\bar{r}_k^*) b_{ki} > \nabla U_{k'}(\bar{r}_{k'}^*) b_{k'i}, \forall k' \neq k\}} \right] \\
&= n \mathbf{E}_{b_{ki}} [b_{ki} \mathbf{Pr}(\nabla U(\bar{r}_k^*) b_{ki} > \nabla U_{k'}(\bar{r}_{k'}^*) b_{k'i}, \forall k' \neq k)] \\
&= n \mathbf{E}_{b_{ki}} \left[b_{ki} \mathbf{Pr} \left(b_{k'i} < \frac{\nabla U(\bar{r}_k^*)}{\nabla U(\bar{r}_{k'}^*)} b_{ki}, \forall k' \neq k \right) \right] \\
&= n \int_0^\infty \eta \prod_{k' \neq k} F_{b_{k'}} \left(\frac{\nabla U(\bar{r}_k^*)}{\nabla U(\bar{r}_{k'}^*)} \eta \right) f_{b_k}(\eta) d\eta \\
&= n \Phi_k(\bar{\mathbf{r}}^*),
\end{aligned} \tag{11}$$

where

$$\Phi_k(\mathbf{r}) = \int_0^\infty \eta \prod_{k' \neq k} F_{b_{k'}} \left(\frac{\nabla U(r_k)}{\nabla U(r_{k'})} \eta \right) f_{b_k}(\eta) d\eta. \tag{12}$$

Notice that $\nabla U(r) > 0$ for all $r > 0$ since $U(r)$ is a continuously increasing function.

From (11), we can see that the optimal throughput $\bar{\mathbf{r}}^*$ is the root of the following nonlinear equation system

$$r_k = n \Phi_k(\mathbf{r}), \quad \forall k = 1, \dots, K. \tag{13}$$

By Brouwer's fixed-point theorem [15], there is at least one root for the system in (13). From (12), we can see that $\Phi_k(\mathbf{r})$ is a decreasing function of r_k , since $\nabla U(r_k)$ decreases with r_k due to the concavity of $U(\cdot)$. Hence, the right-hand-side of the equation in (13) is a continuously decreasing function of r_k , while the left-hand-side is a linearly increasing function of r_k . This means that there is at most one r_k satisfying the equation in (13) given all $\{\bar{r}_{k'}, \forall k' \neq k\}$ are fixed. Therefore, the equation system in (13) has a unique root.

We can find the average throughputs of all users by solving the system of equations in (13). Substituting the optimal average throughputs $\bar{\mathbf{r}}^*$ into (9), we can obtain the optimal DRA policy for each set of users that maximizes the system total utility. Notice that the DRA policy is computed at the beginning of each session, and then the MVNO assigns the SCs to users according to the estimated fast channel fading at the beginning of each time-slot.

C. Optimal System Utility

With the optimal DRA policy, we now derive the analytical expression of the optimal utility $G(\mathcal{X}, n)$, so that the optimal on-demand request problem in (3) can be solved efficiently. With a bit abuse of notation, we use $\bar{\mathbf{r}}^*(n)$ to denote the optimal throughput achieved with n SCs. By solving the equation system (13), we can obtain $\bar{\mathbf{r}}^*(n)$, and the corresponding optimal utility $G(\mathcal{X}, n) = \sum_{k=1}^K U(\bar{r}_k^*(n))$. In general, there is no closed-form expression of $G(\mathcal{X}, n)$. To preserve analytical tractability, we focus on a broad class of utility functions that satisfy (15). Lemma 1 proves that $\bar{\mathbf{r}}^*(n)$ increases linearly with n for such utility functions.

Lemma 1: The average throughput of each user achieved by the optimal DRA policy in (9) increases linearly with the number of SCs, i.e.,

$$\bar{r}_k^*(n) = n\bar{r}_k^*(1), \text{ for } k = 1, \dots, K, \quad (14)$$

if the utility function satisfies the condition

$$\frac{\nabla U(r_1)}{\nabla U(r_2)} = \frac{\nabla U(r_1/n)}{\nabla U(r_2/n)} \quad (15)$$

for any non-negative rates $r_1, r_2 > 0$ and positive integers $n = 1, 2, \dots$.

Proof: With the property in (15), (11) can be written as

$$\bar{r}_k^*(n)/n = \Phi_k(\bar{\mathbf{r}}^*(n)) = \Phi_k(\bar{\mathbf{r}}^*(n)/n), \text{ for } k = 1, \dots, K. \quad (16)$$

This means that $\bar{\mathbf{r}}^*(n)/n$'s for all $n = 1, \dots$ are roots of the equation system $r_k = \Phi_k(\mathbf{r})$ for $k = 1, \dots, K$. As the system has a unique root $\bar{\mathbf{r}}^*(1)$, we have

$$\bar{\mathbf{r}}^*(n)/n = \bar{\mathbf{r}}^*(1), \quad \forall n = 1, \dots, \quad (17)$$

which leads to the proof. ■

The most well-known class of utility functions that satisfies the condition in (15) is the α -fair utility functions [16]

$$U(\bar{\mathbf{r}}) = \begin{cases} \frac{1}{1-\alpha} \bar{\mathbf{r}}^{1-\alpha}, & \text{if } \alpha \geq 0, \alpha \neq 1 \\ \log(\bar{\mathbf{r}}), & \text{if } \alpha = 1, \end{cases} \quad (18)$$

where α is the degree of fairness and $\bar{\mathbf{r}}$ is the average throughput of a user. Specifically, DRA based on α -fair utility turns out to be a throughput maximization problem when $\alpha = 0$, and becomes a delay minimization problem when $\alpha = 2$. Moreover, when $\alpha = 1$, proportional fairness (PF) is achieved among the users with the logarithm utility function. Notice that not

all concave increasing utility functions satisfy the condition in (15), e.g., the exponential utility function where $U(\bar{r}) = 1 - e^{-\bar{r}}$ and the positive diminishing return where $U(\bar{r}) = \ln(1 + \bar{r})$ [17]. In the Section IV and V, we will show that the property in (15) is important in calculating the optimal advance reservation and on-demand request.

With Lemma 1, the optimal utility can be calculated as

$$G(\mathcal{X}, n) = \sum_{k=1}^K U(\bar{r}_k^*(n)) = \sum_{k=1}^K U(n\bar{r}_k^*(1)) \quad (19)$$

We only need to solve the system of equations in (13) once for $\bar{r}^*(1)$, and then the average throughputs with a general n can be obtained accordingly by (14). Notice that $\bar{r}^*(1)$ is uniquely determined by the user locations \mathcal{X} and the distribution of the fast channel fading.

IV. PROPORTIONAL FAIR UTILITY

In this section, we investigate the optimal on-demand request and advance reservation in the proposed two-stage framework when the PF utility function is adopted. The PF utility function is defined by [18]

$$U(\bar{r}) = \log(\bar{r}), \quad (20)$$

for $\bar{r} \geq 0$, which is a special case of α -fair utility functions for $\alpha = 1$. The first-order derivative is given by $\nabla U(\bar{r}) = 1/\bar{r}$, which satisfies the condition in Lemma 1. Therefore, the optimal throughput of each user increases linearly with the number of leased SCs. The corresponding optimal utility can be calculated by

$$\begin{aligned} G(\mathcal{X}, n) &= \sum_{k=1}^K \log(n\bar{r}_k^*(1)) \\ &= K \log(n) + \sum_{k=1}^K \log(\bar{r}_k^*(1)) \end{aligned} \quad (21)$$

From (21), we can see that the total utility increases with $\log(n)$ for a given set of users. Notice that only the first term in (21) contains n , which is related to the two-stage leasing decisions.

A. Optimal On-demand Request

With the optimal DRA and the resulting utility in (21), the on-demand request problem in (3) becomes

$$\max_{n_s \in \mathbb{Z}_+} -c_s n_s + u_g K \log(n_r + n_s) + u_g \sum_{k=1}^K \log(\bar{r}_k^*(1)). \quad (22)$$

Notice that the last term in (22) is irrelevant to n_s . Since the objective is a concave function of n_s , we can obtain the optimal real-value solution by the first-order condition, which equals

$$n_s^* = \max \left(\frac{u_g}{c_s} K - n_r, 0 \right). \quad (23)$$

The optimal integer solution is either the ceiling or the floor of n_s^* , whichever achieves a higher objective function value.

The physical meaning of (23) can be interpreted as follows. The MVNO needs no additional SCs if there are only a small number of active users, i.e., $K \leq n_r c_s / u_g$. Otherwise, the MVNO requests additional SCs, whose number increases linearly with the number of users. Moreover, we can see that the MVNO's demand is proportional to the inverse of c_s , which is the same as the widely used demand curve in telecommunication systems [17]. In addition, we can see that n_s^* is only related to the number of users in a session and is irrelevant to the users' specific locations. This is because that in the expression of $G(\mathcal{X}, n)$ in (21), \mathcal{X} only appears in the second additional term which is irrelevant to n . However, for general utility functions, the optimal on-demand request also depends on \mathcal{X} , which will be shown in Section V.

With the optimal solution in (23), we can compute the corresponding optimal value by

$$Q(\mathcal{X}, c_s, n_r) = u_g \sum_{k=1}^K \log(\bar{r}_k^*(1)) + \begin{cases} c_s n_r - u_g K + u_g K \log(K u_g / c_s), & \text{if } K > n_r c_s / u_g \\ u_g K \log(n_r), & \text{otherwise.} \end{cases} \quad (24)$$

Specially, $Q(\emptyset, c_s, n_r) = 0$, corresponding to zero payoff of an idle session. Here, we use the real-value solution n_s^* as an approximation to the integer solution in calculating $Q(\mathcal{X}, c_s, n_r)$. The approximation error in calculating the optimal advance reservation in the next subsection is negligible, since the error can be averaged out in $\mathbf{E}[Q(\mathcal{X}, c_s, n_r)]$.

B. Optimal Advance Reservation

With the expression of $Q(\mathcal{X}, c_s, n_r)$ in (24), we can find the optimal reservation for a period by solving the stochastic optimization problem in (2). Let $J(n_r)$ denote the objective function in (2). The first-order derivative of $J(n_r)$, denoted by $\nabla J(n_r)$, is given by

$$\begin{aligned} \nabla J(n_r) &= -c_r + \mathbf{E}_{\mathcal{X}, c_s} \left[\frac{\partial Q(\mathcal{X}, c_s, n_r)}{\partial n_r} \right] \\ &= -c_r + \mathbf{E}_{K, c_s} [\min(c_s, u_g K / n_r)] \\ &= -c_r + \mathbf{E}_K [\mathbf{E}_{c_s} [\min(c_s, u_g K / n_r)]] , \end{aligned} \quad (25)$$

where the last step is based on the assumption that K and c_s are independent random variables. With the distribution of c_s , the inner expectation with respect to c_s in (25) can be calculated as

$$\begin{aligned}\mathbf{E}_{c_s} [\min(c_s, c)] &= \int_0^c \eta dF_{c_s}(\eta) + c(1 - F_{c_s}(c)) \\ &= cF_{c_s}(c) - \int_0^c F_{c_s}(\eta) d\eta + c(1 - F_{c_s}(c)) \\ &= \int_0^c (1 - F_{c_s}(\eta)) d\eta.\end{aligned}\tag{26}$$

Since $J(n_r)$ is a concave function of n_r , we can find the optimal real-value solution n_r^* by solving the first order condition

$$\begin{aligned}0 &= -c_r + \mathbf{E}_{K, c_s} [\min(c_s, u_g m / n_r^*)] \\ &= -c_r + \sum_{k=K_a}^{K_b} f_K(k) \int_0^{u_g k / n_r^*} (1 - F_{c_s}(\eta)) d\eta.\end{aligned}\tag{27}$$

The root of (27) can be found numerically using bisection search. The integer solution can be obtained by rounding n_r^* . It can be seen from (27) that n_r^* only depends on the distribution of K instead of the users' specific locations \mathcal{X} .

Proposition 1: With PF utility function, when $\mathbf{E}[c_s] \leq c_r$, the MVNO makes no reservation, i.e., $n_r^* = 0$. In other words, all the spectrum are acquired from the on-demand request in each session.

Proof: From (25), we have

$$\nabla J(n_r) \leq -c_r + \mathbf{E}[c_s],$$

which is less than 0 when $\mathbf{E}[c_s] \leq c_r$. In other words, $J(n_r)$ is a decreasing function of n_r . Hence, the optimal solution is $n_r^* = 0$. ■

Proposition 1 shows that a price discount is essential to motivate the MVNO to place reservation in advance. Moreover, from (27), we can see that n_r^* increases as c_r decreases. This matches the intuition that the MVNO reserves more SCs in advance for a lower reservation price. From the MNO's perspective, advance reservation has advantages of risk-free incomes and simple operation. Hence, the MNO usually sets a discount to encourage MVNOs to reserve resources for a long period of time.

Proposition 2: With PF utility function, the average cost of two-stage leasing increases linearly with the average number of users in each session. That is

$$c_r n_r^* + \mathbf{E}[c_s n_s^*] = u_g \mathbf{E}[K].\tag{28}$$

Proof: From (27), we can compute the cost on the reserved SCs by

$$\begin{aligned} c_r n_r^* &= \mathbf{E}_{K, c_s} [\min(c_s, u_g K / n_r^*)] \times n_r^* \\ &= \mathbf{E}_{K, c_s} [\min(c_s n_r^*, u_g K)]. \end{aligned} \quad (29)$$

Further, from (23), we can calculate the average cost on the on-demand request by

$$\begin{aligned} \mathbf{E}[c_s n_s^*] &= \mathbf{E}_{K, c_s} \left[c_s \times \max \left(\frac{u_g}{c_s} K - n_r^*, 0 \right) \right] \\ &= \mathbf{E}_{K, c_s} [\max(u_g K, c_s n_r^*) - c_s n_r^*] \end{aligned} \quad (30)$$

Adding (29) with (30), we can obtain the total cost by

$$\begin{aligned} c_r n_r^* + \mathbf{E}[c_s n_s^*] &= \mathbf{E}_{K, c_s} [\min(c_s n_r^*, u_g K) + \max(c_s n_r^*, u_g K) - c_s n_r^*] \\ &= \mathbf{E}_{K, c_s} [c_s n_r^* + u_g K - c_s n_r^*] \\ &= u_g \mathbf{E}[K], \end{aligned} \quad (31)$$

and the proof is completed. ■

Proposition 2 shows that more investment is needed for busy period with a large number of users. However, the allocation of the investment into reservation and on-demand request depends on the variation of K and the distribution of c_s . This will be discussed in details with the numerical results in Section VI.

V. EXTENSION TO GENERAL UTILITIES

In this section, we extend the analysis to general utility functions that satisfies the condition in Lemma 1. We will show that in contrast to the case with PF utility in Section IV, the optimal on-demand request and advance reservation depend on both the number of users in each session and their specific locations.

A. Optimal On-demand Request

With the optimal utility from DRA in (19), the on-demand request problem in (3) becomes

$$Q(\mathcal{X}, c_s, n_r) = \max_{n_s \in \mathbb{Z}_+} -c_s n_s + u_g \sum_{k=1}^K U((n_r + n_s) \bar{r}_k^*(1)). \quad (32)$$

By the property in (15), we can obtain the first-order derivative of the objective with respect to n_s as

$$\begin{aligned}
& -c_s + u_g \sum_{k=1}^K \bar{r}_k^*(1) \nabla U((n_r + n_s) \bar{r}_k^*(1)) \\
& = -c_s + u_g \nabla U(n_r + n_s) \sum_{k=1}^K \bar{r}_k^*(1) \frac{\nabla U((n_r + n_s) \bar{r}_k^*(1))}{\nabla U(n_r + n_s)} \\
& = -c_s + u_g \nabla U(n_r + n_s) \sum_{k=1}^K \bar{r}_k^*(1) \frac{\nabla U(\bar{r}_k^*(1))}{\nabla U(1)} \\
& = -c_s + u_g \nabla U(n_r + n_s) \Theta(\mathcal{X}),
\end{aligned} \tag{33}$$

where

$$\Theta(\mathcal{X}) = \frac{1}{\nabla U(1)} \sum_{k=1}^K \bar{r}_k^*(1) \nabla U(\bar{r}_k^*(1)). \tag{34}$$

Notice that $\Theta(\mathcal{X})$ is uniquely determined by \mathcal{X} , and the value can be calculated by solving the system of equations in (13) for $n = 1$. Since the objective in 32 is a concave function of n_s , setting the first-order derivative (33) to zero yields the optimal solution

$$n_s^* = \max \left(\nabla U^{-1} \left(\frac{c_s}{u_g \Theta(\mathcal{X})} \right) - n_r, 0 \right), \tag{35}$$

where $\nabla U^{-1}(\cdot)$ is the inverse function of $\nabla U(\cdot)$. The integer solution can be obtained by rounding n_s^* .

Take general α -fair utility functions as an example. The optimal on-demand request in (35) can be written as

$$n_s^* = \max \left(\left(\frac{u_g \Theta(\mathcal{X})}{c_s} \right)^{1/\alpha} - n_r, 0 \right), \tag{36}$$

where $\Theta(\mathcal{X}) = \sum_{k=1}^K \bar{r}_k^*(1)^{1-\alpha}$. Notice that PF is a special case for $\alpha = 1$ and the results derived in Section IV are consistent with (36). For a general α , n_s^* decreases with higher c_s and larger number of n_r . The impact of \mathcal{X} on n_s^* is not straightforward and will be discussed with the numerical results in Section VI.

Another special case is the linear utility where $U(\bar{r}) = \bar{r}$. In this case, n_s^* is either zero or infinity, depending on $\Theta(\mathcal{X}) = \sum_{k=1}^K \bar{r}_k^*(1)$. This is because that both the leasing cost and the users' utility increase linearly with n_s , and the slope depends on $\Theta(\mathcal{X})$. In practice, the number of SCs for sale is usually limited. Hence, the optimal strategy for the MVNO is to lease as much SCs as possible when $\Theta(\mathcal{X}) > c_s/u_g$.

With the optimal solution in (35), we can obtain the corresponding optimal surplus in the session, which is given by

$$Q(\mathcal{X}, c_s, n_r) = \begin{cases} u_g \sum_{k=1}^K U(n_r \bar{r}_k^*(1)), & \text{if } \Theta(\mathcal{X}) < \frac{c_s}{u_g \nabla U(n_r)}; \\ c_s n_r - c_s \nabla U^{-1} \left(\frac{c_s}{u_g \Theta(\mathcal{X})} \right) + u_g \sum_{k=1}^K U(\nabla U^{-1} \left(\frac{c_s}{u_g \Theta(\mathcal{X})} \right) \bar{r}_k^*(1)), & \text{otherwise.} \end{cases} \quad (37)$$

Here, we also use the real-value solution as an approximation of the integer value solution, since the approximation error will be averaged out in $\mathbf{E}[Q(\mathcal{X}, c_s, n_r)]$ when solving the advance reservation problem.

B. Optimal Advance Reservation

With the expression of $Q(\mathcal{X}, c_s, n_r)$ in (37), we now solve the advance reservation problem in (2). Similar to the notation in Section IV-B, we use $J(n_r)$ and $\nabla J(n_r)$ to denote the objective function of (2) and its first-order derivative respectively. With the condition in (15), we can compute $\nabla J(n_r)$ by

$$\begin{aligned} \nabla J(n_r) &= -c_r + \mathbf{E}_{\mathcal{X}, c_s} [\min(c_s, u_g \nabla U(n_r) \Theta(\mathcal{X}))] \\ &= -c_r + \mathbf{E}_{\mathcal{X}} [\mathbf{E}_{c_s} [\min(c_s, u_g \nabla U(n_r) \Theta(\mathcal{X}))]] \end{aligned} \quad (38)$$

where $\mathbf{E}_{c_s} [\min(c_s, c)]$ is given in (26). Notice that in contrast to the case of PF, $\nabla J(n_r)$ for a general utility depends on the distribution of $\Theta(\mathcal{X})$ instead of the number of users. From (38), we can extend the results in Proposition 1 to general utility functions.

Proposition 3: The MVNO makes no advance reservation and depends on on-demand request only when $\mathbf{E}[c_s] \leq c_r$.

Proof: Similar to the proof of Proposition 1. ■

Due to the lack of closed-form expression of $\Theta(\mathcal{X})$, neither $J(n_r)$ nor $\nabla J(n_r)$ can be computed analytically even with the distribution function of c_s and \mathcal{X} . Instead, we develop a Stochastic Gradient Descent (SGD) algorithm, as presented in Algorithm 1, to find the optimal reservation by off-line sampling. The key idea is to approximate the real gradient by that of a sampled user set in each iteration. By [19, Theorem 3.4], the SGD algorithm returns an ϵ -approximate solution after $T = O(1/\epsilon^2)$ iterations when the step size is set as $\eta_t = 1/\sqrt{t}$.

VI. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we evaluate numerically the performance of the proposed two-stage leasing framework. The users of a tagged MVNO are randomly located inside a cell $\mathcal{D}(o, D)$ with radius

Algorithm 1 SGD for finding the optimal reservation

Input: initial value $n_1 \in \mathbb{Z}_+$, T , step size $\{\eta_t, 1 \leq t \leq T\}$

- 1: **for** $t = 1$ **to** $T - 1$ **do**
- 2: Sample a set of users and their locations $\mathcal{X} = \{x_1, \dots, x_K\}$
- 3: Calculate the average throughputs $\bar{r}^*(1)$ by solving the system in (13) for $n = 1$
- 4: Calculate $\Theta(\mathcal{X})$ by (34)
- 5: Calculate the gradient of the sample with the distribution of c_s

$$d_t = \mathbf{E}_{c_s} [\min(c_s, u_g \nabla U(n_r) \Theta(\mathcal{X}))]$$

- 6: Update and project :

$$\begin{aligned} n_{t+1} &= n_t + \eta_t d_t \\ n_{t+1} &= \max(n_{t+1}, 0) \end{aligned} \tag{39}$$

- 7: **end for**

Output: $n_r^* = \frac{1}{T} \sum_{t=1}^T n_t$

$D = 1$ km and BS at the origin o . The number of users in each session K is an integer random variable uniformly distributed in the interval $[0, 20]$. The wireless channel is Rayleigh faded. A standard path loss model with path loss exponent is 3.67 is adopted [12]. The average received SNR at the cell edge is -6 dB. We consider general α -fair utility functions with $\alpha = 1$ and $\alpha = 0.8$ as two examples. Notice that $\alpha = 1$ corresponds to PF and the results are given in Section IV. For the case with $\alpha = 0.8$, we use Algorithm 1 to calculate n_r^* which converges within 10^6 iterations. Extensive simulations show that the use of other distributions or utility functions does not change the conclusions in this paper, and thus are omitted for brevity.

As we have shown in Section V, the optimal leasing decisions in the two stages depend on the variations of on-demand price c_s and the user traffic \mathcal{X} . In this section, we plan to investigate the impact of the variations of c_s and \mathcal{X} through numerical simulations. In particular, we use the coefficient of variation, as defined in (40), to measure the level of variation of a random variable z .

$$\xi_z = \frac{\sigma_z}{\mu_z}, \tag{40}$$

where σ_z is the standard deviation and μ_z is the mean of the random variable z . With ξ_z , we

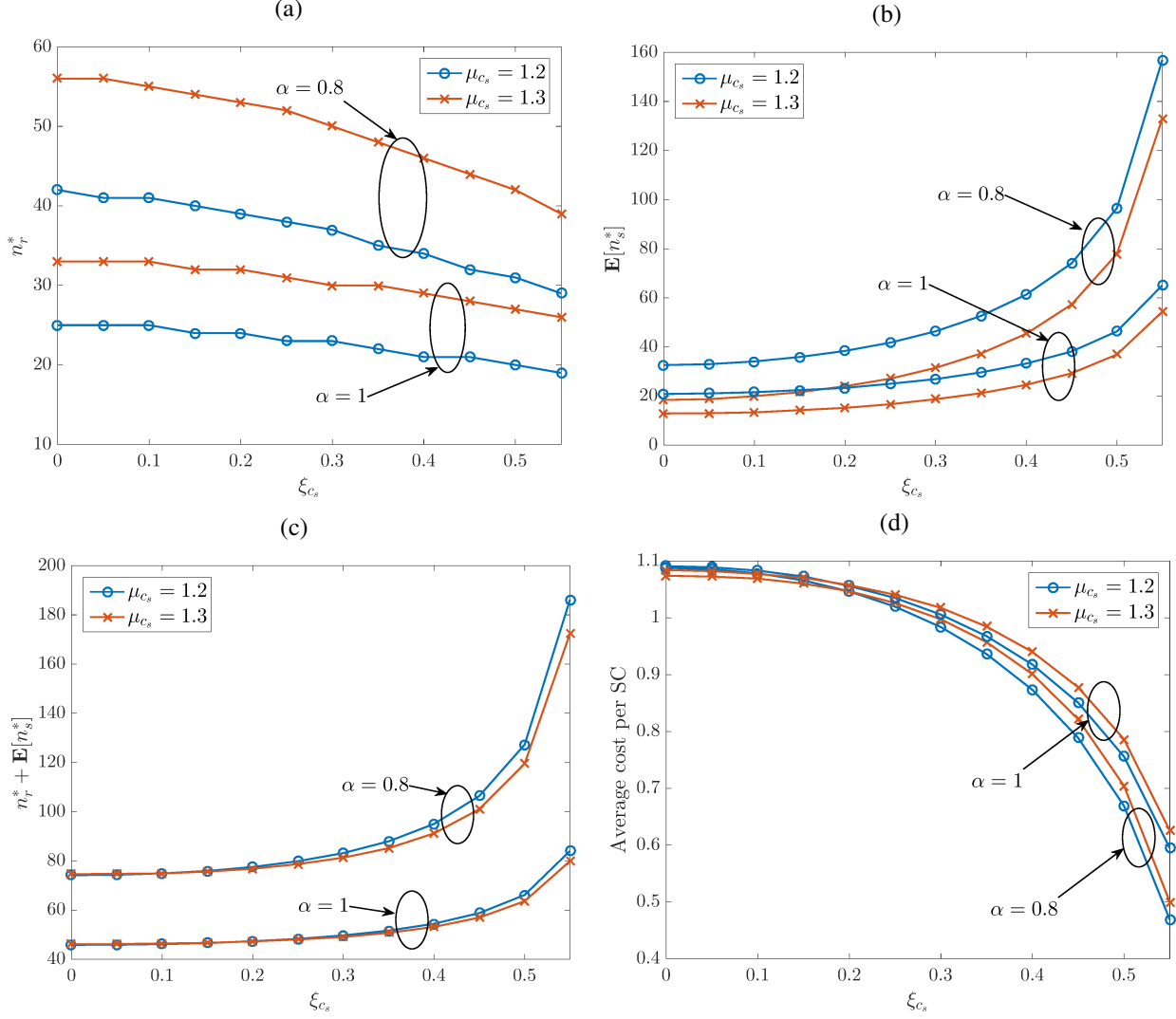


Fig. 3: Optimal two-stage leasing strategy versus on-demand price variation ξ_{c_s} . (a) Optimal advance reservation n_r^* . (b) Expectation of the optimal on-demand request $\mathbf{E}[n_s^*]$. (c) Expected total number of SCs per session $n_r^* + \mathbf{E}[n_s^*]$. (d) Average cost per SC $\frac{c_r n_r^* + \mathbf{E}[c_s n_s^*]}{n_r^* + \mathbf{E}[n_s^*]}$. The mean of the on-demand price μ_{c_s} equals 1.2 or 1.3. The average number of users in a session is $\mu_K = 8$. The fairness factor of the utility function α equals 0.8 or 1.

can compare the variation levels of z under different means.

A. Impact of On-demand Price Variation

In Fig. 3, we fix the mean of the on-demand price μ_{c_s} , and vary the coefficient of variation ξ_{c_s} to investigate the effect of the on-demand price variation on the optimal leasing decisions. Suppose that the reservation price c_r is normalized to 1 and $u_g = 5c_r$. We assume that c_s follows

a uniform distribution. Then, the support of c_s is given by $[(1 - \sqrt{3}\xi_{c_s})\mu_{c_s}, (1 + \sqrt{3}\xi_{c_s})\mu_{c_s}]$. We focus on the case $\mu_{c_s} > c_r$ to avoid the trivial solution $n_r^* = 0$ as shown in Proposition 1 and 3. Specifically, μ_{c_s} is set to be 1.2 and 1.3 in the figures.

In Fig. 3a and 3b, we plot the optimal reservation n_r^* and the expectation of the optimal on-demand request $\mathbb{E}[n_s^*]$, respectively. It can be seen that as ξ_{c_s} becomes larger, n_r^* decreases and $\mathbb{E}[n_s^*]$ increases, meaning that the MVNO makes less reservation in advance and makes more on-demand request when the on-demand price has a larger variation. Moreover, the expectation of the total number of leased SCs, as shown in Fig. 3c, increases as ξ_{c_s} becomes larger. This is because with a larger variance, the lower percentile of c_s becomes lower. Sometimes, realizations of c_s may even be lower than c_r . Thus, the MVNO tends to take the opportunity by leasing more on-demand SCs when c_s is low. Indeed, as shown in Fig. 3d, the average leasing cost per SCs becomes lower when ξ_{c_s} becomes high. As a result, the MNVO tends to lease more SCs in total to increase its surplus.

From Fig. 3, we can see that the observations made above hold for utility functions with different fairness factors α and for different μ_{c_s} . Moreover, for a larger μ_{c_s} , more SCs are reserved in advance and less SCs are leased on demand. The total number of SCs also decreases with μ_{c_s} . This is intuitive, in that when the average on-demand price increases, the MVNO tends to reserve more SCs due to the low cost of advance reservation. The total number of SCs leased is reduced due to the higher average leasing cost.

B. Impact of Traffic Variation

One advantage of the two-stage leasing scheme is that the MVNO has the flexibility of adapting its leasing request according to the real-time traffic realizations during the on-demand request phase. In Fig. 4, we fix the mean of traffic intensity μ_K and vary the coefficient of variation ξ_K to investigate the impact of traffic variation on the optimal leasing decisions. From Fig. 4a and 4b, we can see that when ξ_K increases, the optimal reservation n_r^* decreases, and the expectation of the optimal on-demand request $\mathbb{E}[n_s^*]$ increases. This is intuitive in the sense that the MVNO tends to rely less on advance reservation and more on on-demand request when the real-time traffic demand is more uncertain. This intuition is further confirmed in Fig. 4c, which shows that the proportion of reservation decreases as ξ_K increases. Due to the higher leasing cost in the on-demand stage, the average leasing cost per SC increases with ξ_K , as shown in Fig. 4d. In addition, it can be seen that more SCs are purchased in both reservation and on-demand stages

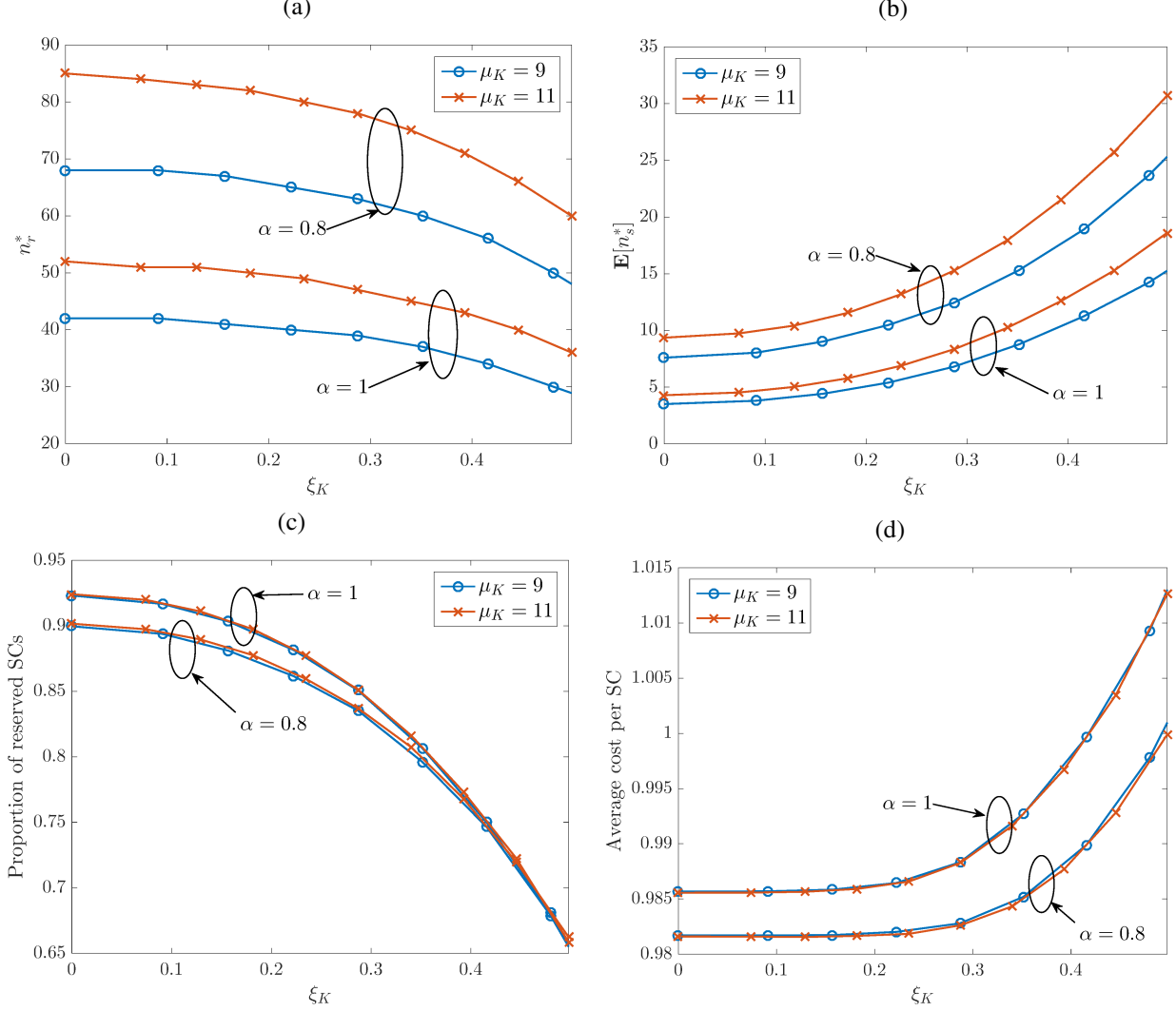


Fig. 4: Optimal two-stage leasing strategies versus traffic variation ξ_K . (a) Optimal advance reservation n_r^* . (b) Expectation of the optimal on-demand request $E[n_s^*]$. (c) Proportion of reserved SCs $\frac{n_r^*}{n_r^* + E[n_s^*]}$. (d) Average cost per SC $\frac{c_r n_r^* + E[c_s n_s^*]}{n_r^* + E[n_s^*]}$. The on-demand price follows a uniform distribution in $[0.7, 1.7]$. The fairness factor of the utility function α equals 0.8 or 1.

for a larger μ_K , which matches our intuition that the MVNO needs more SCs to serve more intense traffic.

C. Comparison with Benchmark Leasing Schemes

For comparison, we consider one-stage spectrum leasing schemes, i.e., reservation-only and on-demand-only, as two benchmarks. In the reservation-only scheme, the MVNO obtains the

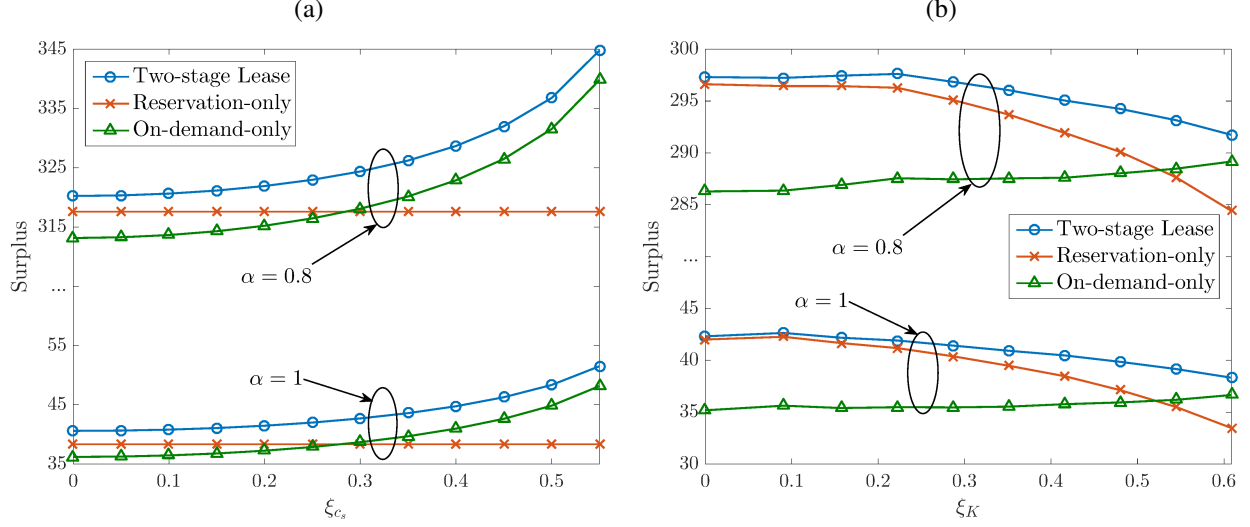


Fig. 5: Comparison of the MVNO's surplus achieved by the optimal two-stage leasing strategy with that of reservation-only and on-demand-only benchmark schemes. (a) Surplus versus on-demand price variation ξ_{c_s} . (b) Surplus versus traffic variation ξ_K . The average number of users in a session is $K = 8$, and the support of K is $[0, 16]$ in (a). The mean of the on-demand price is $\mu_{c_s} = 1.2$, and the support of c_s is $[0.7, 1.7]$ in (b).

SCs at the beginning of a period and has no other chances to purchase spectrum throughout the period. The reservation problem can be formulated similarly to the stochastic programming in (2), except that $Q(\mathcal{X}, c_s, n)$ is replaced by $G(\mathcal{X}, n)$ which is irrelevant to the on-demand price. The optimal reservation n_{ro}^* is then given by

$$n_{ro}^* = \nabla U^{-1} \left(\frac{c_r}{u_g \mathbf{E}_{\mathcal{X}} [\Theta(\mathcal{X})]} \right). \quad (41)$$

The average surplus can be computed correspondingly. In the on-demand-only scheme, the MVNO purchases spectrum dynamically according to the on-demand users and their locations. The on-demand-only problem is a special case of the optimization problem in (3), where $n_r = 0$. Hence, the optimal on-demand request n_{so}^* is given by

$$n_{so}^* = \nabla U^{-1} \left(\frac{c_s}{u_g \Theta(\mathcal{X})} \right). \quad (42)$$

The surplus averaged over the period can be calculated correspondingly.

In Fig. 5, we compare the surplus achieved by the proposed two-stage leasing scheme with the two benchmark schemes. We can see that the two-stage leasing method outperforms both reservation-only and on-demand-only schemes in all cases. Specifically, in Fig. 5a, we plot the

average surplus as a function of the on-demand price variation ξ_{cs} . Since the reservation-only scheme is irrelevant to the on-demand price, the corresponding plot remains constant for different ξ_{cs} . We can see that the surplus achieved by the two-stage leasing scheme is closer to that of the reservation only scheme for small ξ_{cs} , and gradually converges to the on-demand-only scheme for large ξ_{cs} . This implies that the two-stage framework can take advantage of both the discount for advance reservation and the variation of on-demand prices. In Fig. 5b, we plot the average surplus as a function of the traffic intensity variation ξ_K . We can see that the two-stage leasing framework converges to reservation-only as ξ_K becomes small, and converges to on-demand-only when ξ_K becomes large. This implies that the MNO exploits the flexibility of on-demand request to deal with fluctuating traffic, and enjoys low-cost reservation for stable traffic.

VII. CONCLUSIONS

In this paper, we studied a two-stage spectrum leasing problem where the MVNO leases spectrum through both advance reservation and on-demand request. To find the optimal reservation and on-demand request, we formulated the problem as a tri-level stochastic programming problem. Through rigorous analysis, we derived analytical expressions of the optimal values of lower-level problems, which makes the upper-level problems amenable to efficient solution methods. Numerical results showed that the proposed two-stage spectrum leasing framework can take advantage of both the low-cost of advance reservation and the flexibility of on-demand request, and hence increases the surplus of the MVNOs.

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